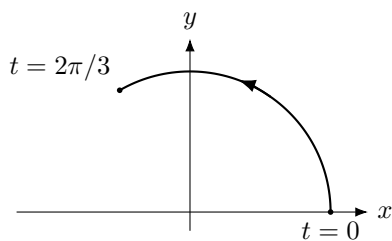


Question 3

A bead moves along

$$x = 3 \cos t, \quad y = 3 \sin t, \quad 0 \leq t \leq \frac{2\pi}{3}.$$

Find the distance traveled.



Question 3 – Solution

Step 1: Differentiate both coordinates.

$$\frac{dx}{dt} = -3 \sin t, \quad \frac{dy}{dt} = 3 \cos t.$$

Step 2: Compute the speed.

$$\begin{aligned} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} &= \sqrt{9 \sin^2 t + 9 \cos^2 t} \\ &= \sqrt{9} = 3. \end{aligned}$$

Step 3: Integrate the speed.

$$\begin{aligned} L &= \int_0^{2\pi/3} 3 \, dt \\ &= [3t]_0^{2\pi/3} = 2\pi. \end{aligned}$$

$L = 2\pi$