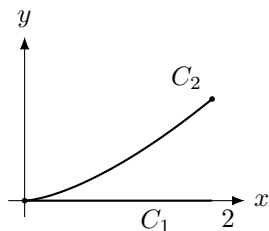


Question 2

For $0 \leq x \leq 2$, compare

$$C_1 : y = 0, \quad C_2 : y = \frac{2}{3}x^{3/2}.$$

Find $L_2 - L_1$.



Question 2 – Solution

Step 1: Find the length of C_1 . Since $y' = 0$,

$$L_1 = \int_0^2 \sqrt{1 + 0^2} \, dx = \int_0^2 1 \, dx = 2.$$

Step 2: Differentiate C_2 .

$$y' = \frac{2}{3} \cdot \frac{3}{2} x^{1/2} = \sqrt{x}.$$

Step 3: Find the length of C_2 .

$$\begin{aligned} L_2 &= \int_0^2 \sqrt{1 + (\sqrt{x})^2} \, dx \\ &= \int_0^2 \sqrt{1 + x} \, dx \\ &= \left[\frac{2}{3} (1 + x)^{3/2} \right]_0^2 \\ &= \frac{2}{3} (3\sqrt{3} - 1) = 2\sqrt{3} - \frac{2}{3}. \end{aligned}$$

Step 4: Subtract.

$$L_2 - L_1 = 2\sqrt{3} - \frac{2}{3} - 2 = 2\sqrt{3} - \frac{8}{3}.$$

$$L_2 - L_1 = 2\sqrt{3} - \frac{8}{3}$$