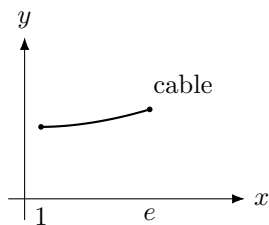


Question 1

A cable follows

$$y = \frac{x^2}{4} - \frac{1}{2} \ln x, \quad 1 \leq x \leq e.$$

Find its exact length.



Question 1 – Solution**Step 1: Differentiate.**

$$y' = \frac{x}{2} - \frac{1}{2x}.$$

Step 2: Simplify the arc-length integrand.

$$\begin{aligned} 1 + (y')^2 &= 1 + \left(\frac{x}{2} - \frac{1}{2x} \right)^2 \\ &= \frac{x^2}{4} + \frac{1}{2} + \frac{1}{4x^2} \\ &= \left(\frac{x}{2} + \frac{1}{2x} \right)^2. \end{aligned}$$

Since $x > 0$,

$$\sqrt{1 + (y')^2} = \frac{x}{2} + \frac{1}{2x}.$$

Step 3: Integrate and evaluate.

$$\begin{aligned} L &= \int_1^e \left(\frac{x}{2} + \frac{1}{2x} \right) dx \\ &= \left[\frac{x^2}{4} + \frac{1}{2} \ln x \right]_1^e \\ &= \left(\frac{e^2}{4} + \frac{1}{2} \right) - \frac{1}{4} = \frac{e^2 + 1}{4}. \end{aligned}$$

$$\boxed{L = \frac{e^2 + 1}{4}}$$