

Question 10

Use a two-sided estimate to determine convergence:

$$\int_1^{\infty} \frac{dx}{x^2 + 3x}.$$

Question 10 – Solution

Step 1: Bound the denominator. For $x \geq 1$, we have $x \leq x^2$, so

$$3x \leq 3x^2.$$

Adding x^2 gives

$$x^2 \leq x^2 + 3x \leq 4x^2.$$

Step 2: Take reciprocals. All terms are positive, so

$$\frac{1}{4x^2} \leq \frac{1}{x^2 + 3x} \leq \frac{1}{x^2}.$$

Step 3: Test the upper comparison.

$$\int_1^{\infty} \frac{dx}{x^2} = 1 < \infty.$$

Step 4: Apply direct comparison. Since the original integrand is nonnegative and bounded above by an integrable function, the integral converges.

converges