

Question 9

Use limit comparison to determine convergence near $x = 0$:

$$\int_0^1 \frac{dx}{\sin x}.$$

Question 9 – Solution

Step 1: Choose a comparison. Since $\sin x \sim x$ near zero, compare with $g(x) = 1/x$.

Step 2: Compute the limit ratio.

$$\begin{aligned} L &= \lim_{x \rightarrow 0^+} \frac{1/\sin x}{1/x} \\ &= \lim_{x \rightarrow 0^+} \frac{x}{\sin x} = 1. \end{aligned}$$

Step 3: Test the comparison integral.

$$\int_0^1 \frac{dx}{x} = \lim_{\varepsilon \rightarrow 0^+} (-\ln \varepsilon) = \infty.$$

Since $0 < L < \infty$, limit comparison shows that the original integral diverges.

diverges