

Question 7

Use dominant powers to determine convergence:

$$\int_1^{\infty} \frac{x^3 + 4}{x^5 + x} dx.$$

Question 7 – Solution

Step 1: Choose a comparison. The dominant powers give

$$\frac{x^3}{x^5} = \frac{1}{x^2},$$

so let $g(x) = 1/x^2$.

Step 2: Compute the limit ratio.

$$\begin{aligned} L &= \lim_{x \rightarrow \infty} \frac{(x^3 + 4)/(x^5 + x)}{1/x^2} \\ &= \lim_{x \rightarrow \infty} \frac{x^5 + 4x^2}{x^5 + x} \\ &= \lim_{x \rightarrow \infty} \frac{1 + 4/x^3}{1 + 1/x^4} = 1. \end{aligned}$$

Step 3: Apply limit comparison. Since $\int_1^\infty x^{-2} dx$ converges and $0 < L < \infty$, the original integral converges.

converges