

Question 5

Use comparison, not integration by parts, to determine convergence:

$$\int_2^{\infty} \frac{\ln x}{x^2} dx.$$

Question 5 – Solution

Step 1: Bound the logarithm. The logarithm grows more slowly than every positive power of x . In particular,

$$\ln x \leq \sqrt{x}$$

for all sufficiently large x .

Step 2: Divide by $x^2 > 0$.

$$0 \leq \frac{\ln x}{x^2} \leq \frac{\sqrt{x}}{x^2} = \frac{1}{x^{3/2}}.$$

Step 3: Test the upper comparison.

$$\int_2^\infty x^{-3/2} dx$$

converges because it is a p -integral with $p = 3/2 > 1$.

Step 4: Apply direct comparison. Therefore, the original integral converges.

converges