

Question 4

Use limit comparison to determine convergence near $x = 0$:

$$\int_0^1 \frac{dx}{\sqrt{x} + x}.$$

Question 4 – Solution

Step 1: Choose a comparison. Near $x = 0$, $\sqrt{x}$ is the dominant denominator term, so use

$$g(x) = \frac{1}{\sqrt{x}} = x^{-1/2}.$$

Step 2: Compute the limit ratio.

$$\begin{aligned} L &= \lim_{x \rightarrow 0^+} \frac{1/(\sqrt{x} + x)}{1/\sqrt{x}} \\ &= \lim_{x \rightarrow 0^+} \frac{\sqrt{x}}{\sqrt{x} + x} \\ &= \lim_{x \rightarrow 0^+} \frac{1}{1 + \sqrt{x}} = 1. \end{aligned}$$

Step 3: Test the comparison integral.

$$\int_0^1 x^{-1/2} dx = 2 < \infty.$$

Since $0 < L < \infty$, limit comparison shows that the original integral converges.

converges