

Question 3

Use a lower comparison to determine convergence:

$$\int_1^{\infty} \frac{2 + \sin x}{x} dx.$$

Question 3 – Solution

Step 1: Bound the numerator. Since $\sin x \geq -1$,

$$2 + \sin x \geq 1.$$

For $x \geq 1$, divide by the positive number x :

$$\frac{2 + \sin x}{x} \geq \frac{1}{x} \geq 0.$$

Step 2: Test the lower comparison.

$$\int_1^\infty \frac{dx}{x} = \lim_{b \rightarrow \infty} \ln b = \infty.$$

Step 3: Apply direct comparison. The smaller integral diverges, so the original integral diverges.

diverges