

Question 2

Use limit comparison to determine convergence:

$$\int_1^{\infty} \frac{3x^2 + 1}{x^4 - 2x + 5} dx.$$

Question 2 – Solution

Step 1: Choose a comparison. The dominant powers give

$$\frac{3x^2}{x^4} = \frac{3}{x^2},$$

so use $g(x) = 1/x^2$.

Step 2: Compute the limit ratio.

$$\begin{aligned} L &= \lim_{x \rightarrow \infty} \frac{(3x^2 + 1)/(x^4 - 2x + 5)}{1/x^2} \\ &= \lim_{x \rightarrow \infty} \frac{3x^4 + x^2}{x^4 - 2x + 5} \\ &= \lim_{x \rightarrow \infty} \frac{3 + 1/x^2}{1 - 2/x^3 + 5/x^4} = 3. \end{aligned}$$

Step 3: Apply limit comparison. Since $0 < L < \infty$ and $\int_1^\infty x^{-2} dx$ converges, the original integral converges.

converges