

Question 1

Use direct comparison to determine convergence:

$$\int_1^{\infty} \frac{dx}{x^2 + \sin^2 x}.$$

Question 1 – Solution

Step 1: Find an upper comparison. Since $\sin^2 x \geq 0$,

$$x^2 + \sin^2 x \geq x^2.$$

Both sides are positive, so taking reciprocals reverses the inequality:

$$0 \leq \frac{1}{x^2 + \sin^2 x} \leq \frac{1}{x^2}.$$

Step 2: Test the comparison integral.

$$\begin{aligned} \int_1^\infty \frac{dx}{x^2} &= \lim_{b \rightarrow \infty} \left[-\frac{1}{x} \right]_1^b \\ &= \lim_{b \rightarrow \infty} \left(1 - \frac{1}{b} \right) = 1. \end{aligned}$$

Step 3: Apply direct comparison. The larger integral converges, so the original integral also converges.

converges