

Question 10

Determine whether the integral converges. If it does, evaluate it.

$$\int_0^{\infty} \frac{dx}{(1+x)^2}$$

Question 10 – Solution

Step 1: Replace infinity with a limit.

$$I = \lim_{b \rightarrow \infty} \int_0^b (1+x)^{-2} dx.$$

Step 2: Integrate.

$$\int (1+x)^{-2} dx = -(1+x)^{-1} = -\frac{1}{1+x}.$$

Step 3: Evaluate the bounds and limit.

$$\begin{aligned} I &= \lim_{b \rightarrow \infty} \left[-\frac{1}{1+x} \right]_0^b \\ &= \lim_{b \rightarrow \infty} \left(-\frac{1}{1+b} + 1 \right) \\ &= 0 + 1 = 1. \end{aligned}$$

1 (convergent)
