

**Question 9**

Find  $c$  so that

$$f(x) = \frac{c}{1+x^2}$$

is a probability density on  $(-\infty, \infty)$ .

**Question 9 – Solution**

**Step 1: Use the normalization requirement.** A probability density must satisfy

$$\int_{-\infty}^{\infty} f(x) dx = 1.$$

Thus

$$1 = c \int_{-\infty}^{\infty} \frac{dx}{1+x^2}.$$

**Step 2: Evaluate the improper integral.** The antiderivative below has finite limits at both  $-\infty$  and  $\infty$ , so both one-sided improper integrals converge separately. Their sum may therefore be computed using symmetric limits.

$$\begin{aligned} \int_{-\infty}^{\infty} \frac{dx}{1+x^2} &= \lim_{R \rightarrow \infty} [\arctan x]_{-R}^R \\ &= \frac{\pi}{2} - \left(-\frac{\pi}{2}\right) = \pi. \end{aligned}$$

**Step 3: Solve for  $c$ .**

$$1 = c\pi \quad \implies \quad c = \frac{1}{\pi}.$$

Because  $c > 0$ ,  $f(x) \geq 0$  for every real  $x$ .

$c = 1/\pi$