

Question 8

Evaluate the improper integral:

$$\int_0^1 \ln x \, dx.$$

Question 8 – Solution

Step 1: Replace the improper endpoint with a limit.

$$I = \lim_{\varepsilon \rightarrow 0^+} \int_{\varepsilon}^1 \ln x \, dx.$$

Step 2: Integrate by parts. Let $u = \ln x$ and $dv = dx$. Then $du = dx/x$ and $v = x$:

$$\begin{aligned} \int \ln x \, dx &= x \ln x - \int x \left(\frac{1}{x} \right) dx \\ &= x \ln x - x. \end{aligned}$$

Step 3: Evaluate the limit.

$$\begin{aligned} I &= \lim_{\varepsilon \rightarrow 0^+} [x \ln x - x]_{\varepsilon}^1 \\ &= \lim_{\varepsilon \rightarrow 0^+} (-1 - \varepsilon \ln \varepsilon + \varepsilon). \end{aligned}$$

Since $\varepsilon \ln \varepsilon \rightarrow 0$ and $\varepsilon \rightarrow 0$,

$$I = -1.$$

$$\boxed{-1}$$