

Question 7

Determine whether the ordinary improper integral converges:

$$\int_{-1}^1 \frac{dx}{x}.$$

Question 7 – Solution

Step 1: Split at the singularity. Ordinary convergence requires both one-sided integrals to converge.

Step 2: Evaluate the left side.

$$\begin{aligned}\int_{-1}^0 \frac{dx}{x} &= \lim_{\varepsilon \rightarrow 0^+} \int_{-1}^{-\varepsilon} \frac{dx}{x} \\ &= \lim_{\varepsilon \rightarrow 0^+} (\ln \varepsilon - \ln 1) = -\infty.\end{aligned}$$

Step 3: Evaluate the right side.

$$\begin{aligned}\int_0^1 \frac{dx}{x} &= \lim_{\varepsilon \rightarrow 0^+} \int_{\varepsilon}^1 \frac{dx}{x} \\ &= \lim_{\varepsilon \rightarrow 0^+} (\ln 1 - \ln \varepsilon) = +\infty.\end{aligned}$$

Since the one-sided integrals diverge, the ordinary integral diverges.

diverges

The symmetric principal value is 0, but it is not an ordinary integral value.