

Question 5

Determine whether the integral converges:

$$\int_2^{\infty} \frac{dx}{x \ln x}.$$

Question 5 – Solution

Step 1: Replace infinity with a limit.

$$I = \lim_{b \rightarrow \infty} \int_2^b \frac{dx}{x \ln x}.$$

Step 2: Substitute $u = \ln x$. Then

$$du = \frac{dx}{x}.$$

The bounds become $u = \ln 2$ and $u = \ln b$, so

$$\begin{aligned} I &= \lim_{b \rightarrow \infty} \int_{\ln 2}^{\ln b} \frac{du}{u} \\ &= \lim_{b \rightarrow \infty} [\ln u]_{\ln 2}^{\ln b} \\ &= \lim_{b \rightarrow \infty} (\ln(\ln b) - \ln(\ln 2)). \end{aligned}$$

Step 3: Evaluate the limit. Since $\ln(\ln b) \rightarrow \infty$, the integral diverges.

diverges