

Question 3

Determine whether the integral converges. If it does, evaluate it.

$$\int_{-1}^1 \frac{dx}{\sqrt{|x|}}$$

Question 3 – Solution

Step 1: Split at the interior singularity.

$$I = \int_{-1}^0 \frac{dx}{\sqrt{|x|}} + \int_0^1 \frac{dx}{\sqrt{|x|}}.$$

The integrand is even, so the two pieces are equal:

$$I = 2 \int_0^1 x^{-1/2} dx.$$

Step 2: Write the improper limit.

$$I = 2 \lim_{\varepsilon \rightarrow 0^+} \int_{\varepsilon}^1 x^{-1/2} dx.$$

Step 3: Integrate and evaluate.

$$\begin{aligned} I &= 2 \lim_{\varepsilon \rightarrow 0^+} [2x^{1/2}]_{\varepsilon}^1 \\ &= 4 \lim_{\varepsilon \rightarrow 0^+} (1 - \sqrt{\varepsilon}) = 4. \end{aligned}$$

$I = 4 \text{ (convergent)}$