

Question 2

Determine whether the integral converges. If it does, evaluate it.

$$\int_0^1 x^{-2/3} dx$$

Question 2 – Solution

Step 1: Identify the improper endpoint. The integrand is undefined at $x = 0$, so replace the lower bound with $\varepsilon > 0$:

$$I = \lim_{\varepsilon \rightarrow 0^+} \int_{\varepsilon}^1 x^{-2/3} dx.$$

Step 2: Integrate.

$$\int x^{-2/3} dx = \frac{x^{1/3}}{1/3} = 3x^{1/3}.$$

Step 3: Evaluate the limit.

$$\begin{aligned} I &= \lim_{\varepsilon \rightarrow 0^+} [3x^{1/3}]_{\varepsilon}^1 \\ &= \lim_{\varepsilon \rightarrow 0^+} (3 - 3\varepsilon^{1/3}) = 3. \end{aligned}$$

3 (convergent)