

**Question 1**

For which real  $p$  does the integral converge? Evaluate it when possible.

$$\int_1^{\infty} x^{-p} dx$$

**Question 1 – Solution**

**Step 1: Replace infinity with a limit.**

$$I = \lim_{b \rightarrow \infty} \int_1^b x^{-p} dx.$$

**Step 2: First suppose  $p \neq 1$ .**

$$\begin{aligned} I &= \lim_{b \rightarrow \infty} \left[ \frac{x^{1-p}}{1-p} \right]_1^b \\ &= \lim_{b \rightarrow \infty} \frac{b^{1-p} - 1}{1-p}. \end{aligned}$$

If  $p > 1$ , then  $1 - p < 0$  and  $b^{1-p} \rightarrow 0$ , so

$$I = \frac{-1}{1-p} = \frac{1}{p-1}.$$

If  $p < 1$ , then  $b^{1-p} \rightarrow \infty$ , so the integral diverges.

**Step 3: Check  $p = 1$  separately.**

$$\int_1^b \frac{dx}{x} = \ln b \rightarrow \infty.$$

|                                                               |
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| $p > 1 : I = \frac{1}{p-1}; \quad p \leq 1 : \text{diverges}$ |
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