

Question 10

Find the integrand whose antiderivative is

$$F(x) = \ln(x^2 + 1) + \arctan x.$$

Then verify your result.

Question 10 – Solution

Step 1: Differentiate the logarithm using the chain rule.

$$\frac{d}{dx} \ln(x^2 + 1) = \frac{2x}{x^2 + 1}.$$

Step 2: Differentiate the inverse tangent.

$$\frac{d}{dx} \arctan x = \frac{1}{x^2 + 1}.$$

Step 3: Add and simplify.

$$\begin{aligned} F'(x) &= \frac{2x}{x^2 + 1} + \frac{1}{x^2 + 1} \\ &= \frac{2x + 1}{x^2 + 1}. \end{aligned}$$

Thus the required integrand is

$$\boxed{f(x) = \frac{2x + 1}{x^2 + 1}}$$

Step 4: Verify by integration.

$$\begin{aligned} \int \frac{2x + 1}{x^2 + 1} dx &= \int \frac{2x}{x^2 + 1} dx + \int \frac{dx}{x^2 + 1} \\ &= \ln(x^2 + 1) + \arctan x + C = F(x) + C. \end{aligned}$$