

Question 9

Choose the best method for each integral. Do not evaluate.

$$\int x^2 e^x dx, \quad \int \frac{dx}{x^2 - 4}, \quad \int \sqrt{9 - x^2} dx, \quad \int \sin^2 x dx.$$

Question 9 – Solution

1. For $\int x^2 e^x dx$, differentiating x^2 repeatedly makes it simpler, while integrating e^x changes nothing. Use **integration by parts** twice.

2. For $\int dx/(x^2 - 4)$, factor

$$x^2 - 4 = (x - 2)(x + 2).$$

The denominator has distinct linear factors, so use **partial fractions**.

3. For $\int \sqrt{9 - x^2} dx$, the form $\sqrt{a^2 - x^2}$ suggests $x = a \sin \theta$. Use **trigonometric substitution** with $x = 3 \sin \theta$.

4. For $\int \sin^2 x dx$, both sine and cosine powers are even. Use the **power-reduction identity**

$$\sin^2 x = \frac{1 - \cos 2x}{2}.$$

$x^2 e^x$	integration by parts
$1/(x^2 - 4)$	partial fractions
$\sqrt{9 - x^2}$	trigonometric substitution
$\sin^2 x$	power reduction