

Question 6

Explain why $u = x^2 + 1$ does not simplify the integral directly, then evaluate:

$$\int \frac{dx}{x^2 + 1}.$$

Question 6 – Solution

Step 1: Test the proposed substitution. If

$$u = x^2 + 1,$$

then

$$du = 2x \, dx.$$

The integral contains dx , but it does not contain the required factor $x \, dx$. Thus the substitution does not produce a direct du factor. One can express dx in terms of u on a branch, but this introduces a square root and is less convenient than the standard antiderivative.

Step 2: Recognize a standard form.

$$\frac{d}{dx}(\arctan x) = \frac{1}{1 + x^2}.$$

Therefore,

$$\int \frac{dx}{x^2 + 1} = \arctan x + C.$$

$$\int \frac{dx}{x^2 + 1} = \arctan x + C$$