

Question 3

Use $u = x^2 + 1$ to evaluate:

$$\int \frac{x^3}{\sqrt{x^2 + 1}} dx.$$

Question 3 – Solution**Step 1: Separate one factor of x .**

$$x^3 dx = x^2(x dx).$$

Step 2: Substitute $u = x^2 + 1$. Then

$$du = 2x dx, \quad x dx = \frac{1}{2} du, \quad x^2 = u - 1.$$

Step 3: Rewrite and simplify.

$$\begin{aligned} I &= \frac{1}{2} \int \frac{u-1}{\sqrt{u}} du \\ &= \frac{1}{2} \int (u^{1/2} - u^{-1/2}) du. \end{aligned}$$

Step 4: Integrate each power.

$$\begin{aligned} I &= \frac{1}{2} \left(\frac{u^{3/2}}{3/2} - \frac{u^{1/2}}{1/2} \right) + C \\ &= \frac{1}{3} u^{3/2} - u^{1/2} + C. \end{aligned}$$

Step 5: Return to x .

$$\frac{1}{3}(x^2 + 1)^{3/2} - \sqrt{x^2 + 1} + C$$