

Question 2

Choose substitution or integration by parts, then evaluate:

$$\int x \ln(1 + x^2) dx.$$

Question 2 – Solution

Step 1: Choose substitution. The expression $1 + x^2$ is inside the logarithm, and its derivative $2x$ matches the remaining factor.

Let

$$u = 1 + x^2, \quad du = 2x \, dx, \quad x \, dx = \frac{1}{2} du.$$

Then

$$I = \frac{1}{2} \int \ln u \, du.$$

Step 2: Integrate $\ln u$ by parts. Choose

$$v = \ln u, \quad dw = du.$$

Then $dv = du/u$ and $w = u$, so

$$\begin{aligned} \int \ln u \, du &= u \ln u - \int u \left(\frac{1}{u} \right) du \\ &= u \ln u - u + C. \end{aligned}$$

Step 3: Restore the factor $1/2$ and return to x .

$$\begin{aligned} I &= \frac{1}{2} (u \ln u - u) + C \\ &= \frac{1}{2} [(1 + x^2) \ln(1 + x^2) - (1 + x^2)] + C. \end{aligned}$$

$$\frac{1}{2} [(1 + x^2) \ln(1 + x^2) - (1 + x^2)] + C$$