

Question 9

Split the numerator, then evaluate:

$$\int \frac{2x + 5}{x^2 + 4x + 8} dx.$$

Question 9 – Solution**Step 1: Split the numerator.**

$$2x + 5 = (2x + 4) + 1.$$

Step 2: Split the integral.

$$I = \int \frac{2x + 4}{x^2 + 4x + 8} dx + \int \frac{dx}{x^2 + 4x + 8}.$$

Step 3: Integrate the derivative term.

$$\int \frac{2x + 4}{x^2 + 4x + 8} dx = \ln(x^2 + 4x + 8).$$

Step 4: Complete the square for the remaining term.

$$x^2 + 4x + 8 = (x + 2)^2 + 4.$$

Therefore,

$$\int \frac{dx}{(x + 2)^2 + 2^2} = \frac{1}{2} \arctan \left(\frac{x + 2}{2} \right).$$

Step 5: Combine the results.

$$\boxed{\ln(x^2 + 4x + 8) + \frac{1}{2} \arctan \frac{x + 2}{2} + C}$$