

Question 8

Determine the real domain and evaluate

$$\int \frac{dx}{\sqrt{6x - x^2}}.$$

Question 8 – Solution

Step 1: Complete the square.

$$\begin{aligned}6x - x^2 &= -(x^2 - 6x) \\&= -((x - 3)^2 - 9) \\&= 9 - (x - 3)^2.\end{aligned}$$

Step 2: Determine the domain. The radicand must be nonnegative:

$$\begin{aligned}9 - (x - 3)^2 &\geq 0, \\(x - 3)^2 &\leq 9, \\-3 &\leq x - 3 \leq 3, \\0 &\leq x \leq 6.\end{aligned}$$

The denominator cannot equal zero, so the integrand's domain is

$$\boxed{0 < x < 6}.$$

Step 3: Substitute $u = x - 3$, so $du = dx$:

$$I = \int \frac{du}{\sqrt{3^2 - u^2}}.$$

Step 4: Use the inverse-sine formula.

$$\begin{aligned}I &= \arcsin\left(\frac{u}{3}\right) + C \\&= \arcsin\left(\frac{x - 3}{3}\right) + C.\end{aligned}$$

$$\boxed{\arcsin \frac{x - 3}{3} + C}$$