

Question 7

Evaluate

$$\int \frac{x^2}{x^2 + 4} dx$$

without partial fractions.

Question 7 – Solution**Step 1: Rewrite the numerator.**

$$x^2 = (x^2 + 4) - 4.$$

Therefore,

$$\begin{aligned}\frac{x^2}{x^2 + 4} &= \frac{(x^2 + 4) - 4}{x^2 + 4} \\ &= 1 - \frac{4}{x^2 + 4}.\end{aligned}$$

Step 2: Split the integral.

$$I = \int 1 \, dx - 4 \int \frac{dx}{x^2 + 2^2}.$$

Step 3: Integrate both terms.

$$\begin{aligned}I &= x - 4 \left[\frac{1}{2} \arctan \left(\frac{x}{2} \right) \right] + C \\ &= x - 2 \arctan \left(\frac{x}{2} \right) + C.\end{aligned}$$

$$x - 2 \arctan \frac{x}{2} + C$$