

Question 6

Find every k that makes $x^2 + kx + 9$ a perfect square. Then integrate

$$\frac{1}{x^2 + kx + 9}.$$

Question 6 – Solution

Step 1: Write the possible perfect squares. Since the constant term is 9, the square must be

$$(x + 3)^2 = x^2 + 6x + 9,$$

$$(x - 3)^2 = x^2 - 6x + 9.$$

Thus

$$k = 6 \quad \text{or} \quad k = -6.$$

Step 2: Integrate when $k = 6$.

$$\begin{aligned} \int \frac{dx}{x^2 + 6x + 9} &= \int \frac{dx}{(x + 3)^2} \\ &= \int (x + 3)^{-2} dx \\ &= -(x + 3)^{-1} + C = -\frac{1}{x + 3} + C. \end{aligned}$$

Step 3: Integrate when $k = -6$.

$$\begin{aligned} \int \frac{dx}{x^2 - 6x + 9} &= \int (x - 3)^{-2} dx \\ &= -(x - 3)^{-1} + C = -\frac{1}{x - 3} + C. \end{aligned}$$

$k = 6 : -\frac{1}{x + 3} + C; \quad k = -6 : -\frac{1}{x - 3} + C$
