

Question 5

Evaluate

$$\int \frac{dx}{\sqrt{x^2 - 6x + 10}}.$$

Question 5 – Solution

Step 1: Complete the square.

$$\begin{aligned}x^2 - 6x + 10 &= (x^2 - 6x + 9) + 1 \\&= (x - 3)^2 + 1.\end{aligned}$$

Step 2: Substitute $u = x - 3$. Then $du = dx$ and

$$I = \int \frac{du}{\sqrt{u^2 + 1}}.$$

Step 3: Use the standard formula.

$$\int \frac{du}{\sqrt{u^2 + a^2}} = \ln \left| u + \sqrt{u^2 + a^2} \right| + C.$$

With $a = 1$,

$$\begin{aligned}I &= \ln \left| u + \sqrt{u^2 + 1} \right| + C \\&= \ln \left| x - 3 + \sqrt{x^2 - 6x + 10} \right| + C.\end{aligned}$$

$$\boxed{\ln |x - 3 + \sqrt{x^2 - 6x + 10}| + C}$$