

Question 4

Find the maximum of $f(x) = 1/(x^2 - 4x + 8)$, then evaluate

$$\int_{-\infty}^{\infty} f(x) \, dx.$$

Question 4 – Solution**Step 1: Complete the square.**

$$x^2 - 4x + 8 = (x - 2)^2 + 4.$$

Step 2: Find the maximum. The denominator is smallest when $(x - 2)^2 = 0$, or $x = 2$. Its minimum is 4, so

$$f(2) = \frac{1}{4}.$$

The antiderivative below has finite limits at both $-\infty$ and ∞ , so both one-sided improper integrals converge separately. Their sum may therefore be computed using symmetric limits. **Step 3: Write the improper integral as a limit.**

$$\begin{aligned} I &= \lim_{R \rightarrow \infty} \int_{-R}^R \frac{dx}{(x - 2)^2 + 2^2} \\ &= \lim_{R \rightarrow \infty} \frac{1}{2} \left[\arctan \left(\frac{x - 2}{2} \right) \right]_{-R}^R. \end{aligned}$$

Step 4: Evaluate the limits.

$$I = \frac{1}{2} \left(\frac{\pi}{2} - \left(-\frac{\pi}{2} \right) \right) = \frac{\pi}{2}.$$

$\max = 1/4, \quad I = \pi/2$
