

Question 3

Evaluate

$$\int \frac{x}{x^2 + 4x + 8} dx.$$

Question 3 – Solution

Step 1: Rewrite the numerator using the derivative of the denominator.

$$x = \frac{1}{2}(2x + 4) - 2.$$

Step 2: Split the integral.

$$I = \frac{1}{2} \int \frac{2x + 4}{x^2 + 4x + 8} dx - 2 \int \frac{dx}{x^2 + 4x + 8}.$$

Step 3: Integrate the derivative term.

$$\frac{1}{2} \int \frac{2x + 4}{x^2 + 4x + 8} dx = \frac{1}{2} \ln(x^2 + 4x + 8).$$

Step 4: Complete the square.

$$x^2 + 4x + 8 = (x + 2)^2 + 4.$$

Therefore,

$$\begin{aligned} -2 \int \frac{dx}{(x + 2)^2 + 2^2} &= -2 \left[\frac{1}{2} \arctan \left(\frac{x + 2}{2} \right) \right] \\ &= -\arctan \left(\frac{x + 2}{2} \right). \end{aligned}$$

Step 5: Combine the results.

$$\boxed{\frac{1}{2} \ln(x^2 + 4x + 8) - \arctan \frac{x + 2}{2} + C}$$