

Question 2

Split the numerator, then evaluate:

$$\int \frac{3x + 1}{x^2 + 2x + 5} dx.$$

Question 2 – Solution

Step 1: Split the numerator using the derivative of the denominator. Since

$$\frac{d}{dx}(x^2 + 2x + 5) = 2x + 2,$$

write

$$3x + 1 = \frac{3}{2}(2x + 2) - 2.$$

Step 2: Split the integral.

$$I = \frac{3}{2} \int \frac{2x + 2}{x^2 + 2x + 5} dx - 2 \int \frac{dx}{x^2 + 2x + 5}.$$

Step 3: Integrate the derivative term.

$$\frac{3}{2} \int \frac{2x + 2}{x^2 + 2x + 5} dx = \frac{3}{2} \ln(x^2 + 2x + 5).$$

Step 4: Complete the square in the remaining term.

$$x^2 + 2x + 5 = (x + 1)^2 + 4.$$

Thus

$$\begin{aligned} -2 \int \frac{dx}{(x + 1)^2 + 2^2} &= -2 \left[\frac{1}{2} \arctan \left(\frac{x + 1}{2} \right) \right] \\ &= -\arctan \left(\frac{x + 1}{2} \right). \end{aligned}$$

Step 5: Combine the results.

$$\boxed{\frac{3}{2} \ln(x^2 + 2x + 5) - \arctan \frac{x + 1}{2} + C}$$