

Question 1

Complete the square and evaluate

$$\int \frac{dx}{x^2 + 6x + 13}.$$

Question 1 – Solution

Step 1: Complete the square.

$$\begin{aligned}x^2 + 6x + 13 &= (x^2 + 6x + 9) + 4 \\&= (x + 3)^2 + 4.\end{aligned}$$

Step 2: Rewrite the integral.

$$I = \int \frac{dx}{(x + 3)^2 + 2^2}.$$

Step 3: Use the arctangent formula. Let $u = x + 3$, so $du = dx$:

$$\begin{aligned}I &= \int \frac{du}{u^2 + 2^2} \\&= \frac{1}{2} \arctan\left(\frac{u}{2}\right) + C \\&= \frac{1}{2} \arctan\left(\frac{x + 3}{2}\right) + C.\end{aligned}$$

$$\boxed{\frac{1}{2} \arctan \frac{x + 3}{2} + C}$$