

Question 10

Use one substitution to evaluate:

$$\int \frac{x+1}{\sqrt{x^2+2x+5}} dx.$$

Question 10 – Solution

Step 1: Substitute the expression inside the square root.

$$u = x^2 + 2x + 5.$$

Differentiate:

$$du = (2x + 2) dx = 2(x + 1) dx, \quad (x + 1) dx = \frac{1}{2} du.$$

Step 2: Rewrite the integral.

$$\begin{aligned} I &= \int \frac{x + 1}{\sqrt{x^2 + 2x + 5}} dx \\ &= \frac{1}{2} \int u^{-1/2} du. \end{aligned}$$

Step 3: Integrate and return to x .

$$\begin{aligned} I &= \frac{1}{2} \left(\frac{u^{1/2}}{1/2} \right) + C \\ &= u^{1/2} + C \\ &= \sqrt{x^2 + 2x + 5} + C. \end{aligned}$$

$\sqrt{x^2 + 2x + 5} + C$