

Question 9

Differentiate $F(x)$, then identify the substitution used to integrate $F'(x)$:

$$F(x) = \frac{2}{3}(1 + x^2)^{3/2}.$$

Question 9 – Solution

Step 1: Differentiate with the chain rule.

$$\begin{aligned} F'(x) &= \frac{2}{3} \cdot \frac{3}{2} (1+x^2)^{1/2} \cdot (2x) \\ &= 2x\sqrt{1+x^2}. \end{aligned}$$

Step 2: Reverse the process by integration.

$$\int 2x\sqrt{1+x^2} \, dx.$$

Let

$$u = 1 + x^2, \quad du = 2x \, dx.$$

Then

$$\begin{aligned} \int 2x\sqrt{1+x^2} \, dx &= \int u^{1/2} \, du \\ &= \frac{u^{3/2}}{3/2} + C \\ &= \frac{2}{3}(1+x^2)^{3/2} + C. \end{aligned}$$

$$\boxed{\int 2x\sqrt{1+x^2} \, dx = F(x) + C}$$