

Question 6

Find $a > 0$ so

$$\int_0^a \sqrt{x} \, dx = 18.$$

Question 6 – Solution

Step 1: Evaluate the definite integral.

$$\begin{aligned}\int_0^a \sqrt{x} \, dx &= \int_0^a x^{1/2} \, dx \\ &= \left[\frac{x^{3/2}}{3/2} \right]_0^a \\ &= \left[\frac{2}{3} x^{3/2} \right]_0^a = \frac{2}{3} a^{3/2}.\end{aligned}$$

Step 2: Set the result equal to 18 and solve.

$$\begin{aligned}\frac{2}{3} a^{3/2} &= 18, \\ a^{3/2} &= 27, \\ \left(a^{3/2} \right)^{2/3} &= 27^{2/3}, \\ a &= (\sqrt[3]{27})^2 = 3^2 = 9.\end{aligned}$$

$a = 9$