

Question 5

Evaluate

$$\int \frac{\sqrt{x}}{1+x} dx.$$

Question 5 – Solution

Step 1: Substitute $u = \sqrt{x}$. Then

$$x = u^2, \quad dx = 2u \, du.$$

Step 2: Rewrite the integral.

$$\begin{aligned} \int \frac{\sqrt{x}}{1+x} dx &= \int \frac{u}{1+u^2} (2u \, du) \\ &= 2 \int \frac{u^2}{1+u^2} du. \end{aligned}$$

Step 3: Divide the numerator.

$$\frac{u^2}{1+u^2} = \frac{(1+u^2) - 1}{1+u^2} = 1 - \frac{1}{1+u^2}.$$

Step 4: Integrate and return to x .

$$\begin{aligned} I &= 2 \int \left(1 - \frac{1}{1+u^2} \right) du \\ &= 2u - 2 \arctan u + C \\ &= 2\sqrt{x} - 2 \arctan \sqrt{x} + C. \end{aligned}$$

$$\boxed{2\sqrt{x} - 2 \arctan \sqrt{x} + C}$$