

Question 4

Rationalize the integrand and evaluate

$$\int \frac{dx}{\sqrt{x+1} - \sqrt{x}}.$$

Question 4 – Solution

Step 1: Multiply by the conjugate.

$$\begin{aligned}\frac{1}{\sqrt{x+1}-\sqrt{x}} &= \frac{1}{\sqrt{x+1}-\sqrt{x}} \cdot \frac{\sqrt{x+1}+\sqrt{x}}{\sqrt{x+1}+\sqrt{x}} \\ &= \frac{\sqrt{x+1}+\sqrt{x}}{(x+1)-x} \\ &= \sqrt{x+1}+\sqrt{x}.\end{aligned}$$

Step 2: Integrate both terms.

$$\begin{aligned}I &= \int (x+1)^{1/2} dx + \int x^{1/2} dx \\ &= \frac{(x+1)^{3/2}}{3/2} + \frac{x^{3/2}}{3/2} + C \\ &= \frac{2}{3}(x+1)^{3/2} + \frac{2}{3}x^{3/2} + C.\end{aligned}$$

$$\boxed{\frac{2}{3}(x+1)^{3/2} + \frac{2}{3}x^{3/2} + C}$$