

Question 3

Find the geometric area between $y = \sqrt[3]{x}$ and the x -axis on $[-1, 8]$.

Question 3 – Solution

Step 1: Split at the x -intercept. The function is negative on $[-1, 0]$ and positive on $[0, 8]$, so

$$A = - \int_{-1}^0 x^{1/3} dx + \int_0^8 x^{1/3} dx.$$

Step 2: Find an antiderivative.

$$\int x^{1/3} dx = \frac{x^{4/3}}{4/3} = \frac{3}{4}x^{4/3}.$$

Step 3: Evaluate both pieces.

$$\begin{aligned} A &= - \left[\frac{3}{4}x^{4/3} \right]_{-1}^0 + \left[\frac{3}{4}x^{4/3} \right]_0^8 \\ &= - \left(0 - \frac{3}{4} \right) + \left(\frac{3}{4} \cdot 16 - 0 \right) \\ &= \frac{3}{4} + 12 = \frac{51}{4}. \end{aligned}$$

$$\boxed{A = \frac{51}{4}}$$