

Question 1

Use $u = \sqrt{x}$ to evaluate

$$\int \frac{dx}{1 + \sqrt{x}}.$$

Question 1 – Solution

Step 1: Substitute $u = \sqrt{x}$. Then

$$x = u^2, \quad dx = 2u \, du.$$

Step 2: Rewrite the integral.

$$\int \frac{dx}{1 + \sqrt{x}} = \int \frac{2u}{1 + u} \, du.$$

Step 3: Divide the numerator.

$$\frac{u}{u+1} = \frac{(u+1) - 1}{u+1} = 1 - \frac{1}{u+1}.$$

Step 4: Integrate and return to x .

$$\begin{aligned} 2 \int \left(1 - \frac{1}{u+1} \right) du &= 2u - 2 \ln |u+1| + C \\ &= 2\sqrt{x} - 2 \ln(1 + \sqrt{x}) + C. \end{aligned}$$

$$\boxed{2\sqrt{x} - 2 \ln(1 + \sqrt{x}) + C}$$