

Question 9

Use partial fractions to evaluate:

$$\int \frac{dx}{x^3 - x^2} = \int \frac{dx}{x^2(x - 1)}.$$

Question 9 – Solution

Step 1: Set up partial fractions. The repeated factor x^2 requires both $1/x$ and $1/x^2$:

$$\frac{1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}.$$

Step 2: Clear the denominators.

$$1 = Ax(x-1) + B(x-1) + Cx^2.$$

Step 3: Find B and C .

$$x = 0: \quad 1 = -B \implies B = -1,$$

$$x = 1: \quad 1 = C \implies C = 1.$$

Step 4: Find A by setting $x = 2$.

$$\begin{aligned} 1 &= A(2)(1) + (-1)(1) + (1)(4) \\ &= 2A + 3, \\ -2 &= 2A \implies A = -1. \end{aligned}$$

Step 5: Substitute and integrate each term.

$$\begin{aligned} I &= \int \left(-\frac{1}{x} - \frac{1}{x^2} + \frac{1}{x-1} \right) dx \\ &= -\int \frac{dx}{x} - \int x^{-2} dx + \int \frac{dx}{x-1} \\ &= -\ln|x| - \frac{x^{-1}}{-1} + \ln|x-1| + C \\ &= -\ln|x| + \frac{1}{x} + \ln|x-1| + C. \end{aligned}$$

Therefore,

$$I = -\ln|x| + \frac{1}{x} + \ln|x-1| + C.$$