

Question 8

Find A , B , and C in the decomposition

$$\frac{1}{x(x-2)(x+3)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+3}.$$

Use $x = 0$, 2 , and -3 . State (A, B, C) .

Question 8 – Solution

Step 1: Start with the decomposition.

$$\frac{1}{x(x-2)(x+3)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+3}$$

Step 2: Multiply every term by $x(x-2)(x+3)$.

$$1 = A(x-2)(x+3) + Bx(x+3) + Cx(x-2).$$

Step 3: Use values that eliminate two terms at a time.

$$\begin{array}{llll} x = 0 : & 1 = -6A & \implies & A = -\frac{1}{6}, \\ x = 2 : & 1 = 10B & \implies & B = \frac{1}{10}, \\ x = -3 : & 1 = 15C & \implies & C = \frac{1}{15}. \end{array}$$

Step 4: State the ordered triple.

$$(A, B, C) = \left(-\frac{1}{6}, \frac{1}{10}, \frac{1}{15} \right).$$