

Question 6

A student proposes the decomposition

$$\frac{1}{(x^2 + 1)^2} = \frac{A}{x^2 + 1} + \frac{B}{(x^2 + 1)^2}.$$

1. Why is this decomposition not useful? Give the correct general form.
2. Use $x = \tan \theta$ to evaluate:

$$\int \frac{dx}{(x^2 + 1)^2}$$

Question 6 – Solution**Part 1: Check the decomposition.**

Each power of an irreducible quadratic needs a linear numerator. The general form is

$$\frac{Ax + B}{x^2 + 1} + \frac{Cx + D}{(x^2 + 1)^2}.$$

The original fraction already has this form: $A = B = C = 0$ and $D = 1$. Therefore, decomposing it only reproduces the original integrand and does not make the integral easier.

Part 2: Evaluate the integral.

Step 1: Substitute $x = \tan \theta$. Then

$$dx = \sec^2 \theta \, d\theta, \quad x^2 + 1 = \tan^2 \theta + 1 = \sec^2 \theta.$$

Step 2: Simplify and integrate.

$$\begin{aligned} I &= \int \frac{\sec^2 \theta}{\sec^4 \theta} d\theta = \int \cos^2 \theta \, d\theta \\ &= \frac{1}{2} \int (1 + \cos 2\theta) \, d\theta \\ &= \frac{\theta}{2} + \frac{\sin 2\theta}{4} + C. \end{aligned}$$

Step 3: Return to x .

$$\theta = \arctan x, \quad \sin 2\theta = \frac{2 \tan \theta}{1 + \tan^2 \theta} = \frac{2x}{1 + x^2}.$$

Substitute both expressions:

$$I = \frac{x}{2(x^2 + 1)} + \frac{1}{2} \arctan x + C.$$