

Question 5

Find the value of k that removes the $\ln|x+1|$ term. Then evaluate:

$$\int \frac{kx+1}{x^2-1} dx.$$

Question 5 – Solution**Step 1: Set up the decomposition.**

$$\frac{kx+1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}.$$

Step 2: Clear the denominators.

$$kx+1 = A(x+1) + B(x-1).$$

Step 3: Find the coefficients.

$$\begin{aligned} x=1: \quad k+1 &= 2A \implies A = \frac{k+1}{2}, \\ x=-1: \quad -k+1 &= -2B \implies B = \frac{k-1}{2}. \end{aligned}$$

Step 4: Remove the $\ln|x+1|$ term. Since integrating $B/(x+1)$ gives $B \ln|x+1|$, set $B=0$:

$$B=0 \implies \frac{k-1}{2} = 0 \implies \boxed{k=1}.$$

Step 5: Substitute $k=1$, simplify, and integrate.

$$\frac{x+1}{x^2-1} = \frac{x+1}{(x-1)(x+1)} = \frac{1}{x-1}, \quad x \neq \pm 1.$$

$$\int \frac{x+1}{x^2-1} dx = \int \frac{dx}{x-1} = \boxed{\ln|x-1| + C}.$$