

Question 3

Decompose into partial fractions. Include every power of $(x - 1)$:

$$\frac{2x + 3}{x(x - 1)^2}.$$

Do not integrate.

Question 3 – Solution

Step 1: Include a term for every factor and repeated power.

$$\frac{2x+3}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}.$$

Step 2: Clear the denominators.

$$2x+3 = A(x-1)^2 + Bx(x-1) + Cx.$$

Step 3: Find A and C .

$$x=0: \quad 3 = A(-1)^2 \implies A=3,$$

$$x=1: \quad 5 = C(1) \implies C=5.$$

Step 4: Find B by using $x=2$.

$$\begin{aligned} 7 &= 3(1)^2 + B(2)(1) + 5(2) \\ &= 3 + 2B + 10, \\ -6 &= 2B \implies B = -3. \end{aligned}$$

Step 5: State the result.

$$\boxed{\frac{2x+3}{x(x-1)^2} = \frac{3}{x} - \frac{3}{x-1} + \frac{5}{(x-1)^2}},$$

and the requested coefficients are

$$\boxed{A=3, \quad B=-3, \quad C=5}.$$