

Question 2

Use polynomial division, then evaluate:

$$\int \frac{x^3 + 1}{x^2 - 1} dx.$$

State the excluded values of x .

Question 2 – Solution

Step 1: Divide the polynomials. Since $x^3/x^2 = x$,

$$\begin{aligned}x(x^2 - 1) &= x^3 - x, \\(x^3 + 1) - (x^3 - x) &= x + 1.\end{aligned}$$

Thus $x^3 + 1 = x(x^2 - 1) + (x + 1)$, so

$$\frac{x^3 + 1}{x^2 - 1} = x + \frac{x + 1}{x^2 - 1}.$$

Step 2: Factor and simplify.

$$\begin{aligned}\frac{x^3 + 1}{x^2 - 1} &= x + \frac{x + 1}{(x - 1)(x + 1)} \\&= x + \frac{1}{x - 1},\end{aligned}$$

because the original denominator is zero at both $x = -1$ and $x = 1$.

Step 3: Integrate term by term.

$$\begin{aligned}\int \frac{x^3 + 1}{x^2 - 1} dx &= \int x dx + \int \frac{dx}{x - 1} \\&= \boxed{\frac{x^2}{2} + \ln |x - 1| + C}, \quad x \neq \pm 1.\end{aligned}$$