

Question 1

Find A and B , then evaluate the integral:

$$\frac{7x+1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}.$$

$$\int \frac{7x+1}{(x-1)(x+2)} dx.$$

Question 1 – Solution

Step 1: Set up the decomposition.

$$\frac{7x+1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}.$$

Step 2: Clear the denominators. Multiply by $(x-1)(x+2)$:

$$7x+1 = A(x+2) + B(x-1).$$

Step 3: Find the coefficients.

$$\begin{aligned} x=1: \quad 8 &= 3A \quad \implies \quad A = \frac{8}{3}, \\ x=-2: \quad -13 &= -3B \quad \implies \quad B = \frac{13}{3}. \end{aligned}$$

Step 4: Substitute and integrate each term.

$$\begin{aligned} \int \frac{7x+1}{(x-1)(x+2)} dx &= \int \left(\frac{8/3}{x-1} + \frac{13/3}{x+2} \right) dx \\ &= \frac{8}{3} \int \frac{dx}{x-1} + \frac{13}{3} \int \frac{dx}{x+2} \\ &= \boxed{\frac{8}{3} \ln|x-1| + \frac{13}{3} \ln|x+2| + C}. \end{aligned}$$