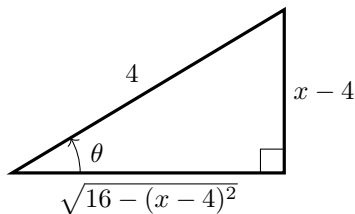


Question 5

Complete the square, choose an appropriate trigonometric substitution, and evaluate

$$\int \frac{dx}{\sqrt{8x - x^2}}.$$

Question 5 – Solution



Complete the square inside the radical:

$$\begin{aligned} 8x - x^2 &= -(x^2 - 8x) \\ &= -((x - 4)^2 - 16) \\ &= 16 - (x - 4)^2. \end{aligned}$$

Let

$$x - 4 = 4 \sin \theta, \quad dx = 4 \cos \theta \, d\theta.$$

Choose $-\pi/2 < \theta < \pi/2$, so $\cos \theta > 0$. Then

$$\begin{aligned} \sqrt{16 - (x - 4)^2} &= \sqrt{16 - 16 \sin^2 \theta} \\ &= 4 \cos \theta. \end{aligned}$$

Thus,

$$\begin{aligned} I &= \int \frac{4 \cos \theta \, d\theta}{4 \cos \theta} \\ &= \int d\theta = \theta + C. \end{aligned}$$

Since $\sin \theta = (x - 4)/4$,

$$\boxed{I = \arcsin\left(\frac{x - 4}{4}\right) + C.}$$

This formula applies on intervals inside $0 < x < 8$, where the radical is real and nonzero.