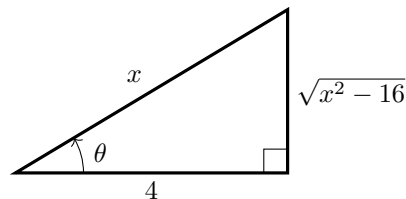


**Question 4**

Evaluate using a trigonometric substitution:

$$\int \frac{dx}{x^2 \sqrt{x^2 - 16}}, \quad x > 4.$$

## Question 4 – Solution



Because the radical has the form  $\sqrt{x^2 - 16}$ , let

$$x = 4 \sec \theta, \quad dx = 4 \sec \theta \tan \theta \, d\theta.$$

Since  $x > 4$ , take  $0 < \theta < \pi/2$ . Then

$$\sqrt{x^2 - 16} = \sqrt{16 \sec^2 \theta - 16} = 4 \tan \theta.$$

Therefore,

$$\begin{aligned} I &= \int \frac{4 \sec \theta \tan \theta \, d\theta}{(16 \sec^2 \theta)(4 \tan \theta)} \\ &= \frac{1}{16} \int \cos \theta \, d\theta \\ &= \frac{1}{16} \sin \theta + C. \end{aligned}$$

From the reference triangle,

$$\sec \theta = \frac{x}{4}, \quad \sin \theta = \frac{\sqrt{x^2 - 16}}{x}.$$

Hence,

$$\boxed{I = \frac{\sqrt{x^2 - 16}}{16x} + C}.$$

Differentiation recovers  $1/(x^2 \sqrt{x^2 - 16})$ .