

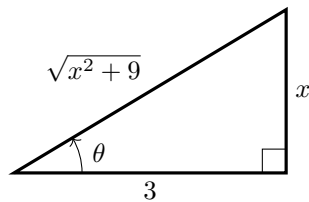
Question 3

Find the total area under

$$y = (x^2 + 9)^{-3/2}$$

on $0 \leq x < \infty$.

Question 3 – Solution



The required area is the improper integral

$$I = \lim_{b \rightarrow \infty} \int_0^b \frac{dx}{(x^2 + 9)^{3/2}}.$$

Let $x = 3 \tan \theta$. Then

$$dx = 3 \sec^2 \theta \, d\theta, \quad (x^2 + 9)^{3/2} = 27 \sec^3 \theta.$$

The bounds transform as $x = 0 \Rightarrow \theta = 0$ and $x \rightarrow \infty \Rightarrow \theta \rightarrow \pi/2$. Thus,

$$\begin{aligned} I &= \frac{1}{9} \int_0^{\pi/2} \cos \theta \, d\theta \\ &= \frac{1}{9} [\sin \theta]_0^{\pi/2} \\ &= \frac{1}{9}. \end{aligned}$$

The integrand is positive, so this convergent improper integral equals the geometric area. Therefore,

$$\boxed{I = \frac{1}{9}}.$$